ECON 2123 Introduction of Econometrics
HW3 Answer Key

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Topics: Stock and Watson Chapter 5, Hypothesis Testing of the Coefficients (105 pts)

Section 1, Question 1. (S&W Q5.1, 20 pts)
Note: Different versions of textbook might have different numbers in the questions, as long as the calculation is correct, you will get the full credits.

2.1 (a) The 95% confidence interval for β_1 is $[-5.82 \pm 1.96 \times 2.21]$, that is

$$-10.152 \leq \beta_1 \leq 1.4884$$

.
(b) Calculate the t-statistic:

$$t^{act} = \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} = \frac{-5.82}{2.21} = -2.6335$$

The p-value for the test $H_o : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$

$$\mathrm{p-value} = 2\Phi(-|\mathrm{t^{act}}|) = 2\Phi(-2.6335) = 2 \times 0.0042 = 0.0084.$$

The p-value is less than 0.01, so we can reject the null hypothesis at the 5% significance level, and also at the 1% significance level. Function $\Phi()$ denotes the CDF (Cumulative Distribution Function) of the Standard Normal Distribution. Since we have large sample size, the distribution that t-statistic follows will approximate standard normal. This number is not directly computable, instead, it could be found in the corresponding distribution table. i.e. the appendix in Stock and Waston.

(c) The t-statistic under the new null hypothesis is

$$t^{act} = \frac{\hat{\beta}_1 - (-5.6)}{SE(\hat{\beta}_1)} = \frac{-0.22}{2.21} = -0.1$$

The p-value for the test $H_o : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$

$$\mathrm{p-value} = 2\Phi(-|\mathrm{t^{act}}|) = 2\Phi(-0.1) = 0.92.$$

The p-value is larger than 0.10, so we cannot reject the null hypothesis at the 10%, 5% or 1% significance level. Because $\beta_1 = -5.6$ is not rejected at the 5% level, this value is contained in the 95% confidence interval.

(d) The 99% confidence interval for β_1 is $[520.44 \pm 2.58 \times 20.4]$, that is,

$$467.7 \leq \beta_1 \leq 573.0$$

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Section 1, Question 2. (S&W Q5.2, 20 pts)
Note: Different versions of textbook might have different numbers in the questions, as long as the calculation is correct, you will get the full credits.

5.2 (a) The estimated gender gap equals \$2.12/hour.

(b) The null and alternative hypotheses are $H_o : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$.

The t-statistic is

$$t^{act} = \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} = \frac{2.12}{0.36} = 5.89$$

(c) And then, the p-value for the test is

$$\text{p} - \text{value} = 2\Phi(-|t^{act}|) = 2 \times \Phi(-5.89) = 2 \times 0.0000 = 0.000$$

(round to fourth decimal places)

The p-value is less than 0.01, so we can reject the null hypothesis that there is no gender gap at a 1% significance level. The 95% confidence interval for the gender gap is $(2.12 \pm 1.96 \times 0.36)$, that is,

$$1.415 \leq \beta_0 \leq 2.83$$

(d) The sample average wage of women is $\hat{\beta}_0 = 12.52$/hour. The sample average wage of men is

$$\hat{\beta}_0 + \hat{\beta}_1 = \$12.52 + \$2.12 = \$14.64/\text{hour}.$$

Section 2, Question 1. (Stata Outputs, 30 pts)

1) Identify the standard error of $\hat{\beta}_1$: 0.0592205
2) Calculate the t value of $\hat{\beta}_1$

$$t = (\frac{-.1689301 - 0}{.0592205}) = -2.85256119 \quad (-2.85 \text{ rounding to two decimal places})$$

3) Test the null hypothesis $\hat{\beta}_1 = 0$ against the alternative hypothesis that $\hat{\beta}_1$ different from zero

$|t| = |-2.85256119| > 1.96$, thus we can reject the null hypothesis and accept the alternative hypothesis. Conclude that $\hat{\beta}_1$ is significantly different from zero.

4) Create a 95 % confidence interval for β_1

$$(-0.1689301 \pm 1.96 \times .0592205)$$

$$[-.1689301 - (1.96 \times .0592205), -.1689301 + (1.96 \times .0592205)]$$

$$[-0.28500228, -0.05285792]$$

zero ($H_0 : \beta = 0$) is not within confidence interval (reject H_o and accept H_1)
5) Write your equations including standard errors

$$Educ1 = \underset{(0.38)}{4.34} - \underset{(0.059)}{0.17} \times hhsize$$

6) Explain the meaning of the slope. Increasing household size by one unit decreases the education of the head of the household by 0.17 units.

Section 2, Question 2. (Stata Outputs, 30 pts)

1) Identify the standard error of $\hat{\beta}_1$: 0.1832509

2) Calculate the t-statistics (t-value) of $\hat{\beta}_1$

$$t = \frac{-0.0157957}{0.1832509} = -0.0862 \quad (-0.09 \text{ rounding to two decimal places})$$

3) Test the null hypothesis ($H_o : \beta = 0$) against the alternative hypothesis that β is different from zero ($H_1 : \beta \neq 0$).

Since we already calculate out the t-statistics, then we can compare it with the critical value under the 95% confidence interval of two-tail test.

$$|-0.0862| = 0.0862 < 1.96,$$

Hence, we fail to reject (i.e. accept) the null hypothesis that $\beta_1 = 0$ and reject the alternative hypothesis H_1.

4) Create a 95 % confidence interval for $\hat{\beta}_1$

$$-.0157957 \pm 1.96 \times .1832509 = (-0.37497, 0.343376)$$

Since, zero (critical value under null hypothesis $H_o : \beta_1 = 0$) is within this range,
we fail to reject H_o and reject H_1.

5) Write your equations including standard errors

$$\text{grrv} = \underset{(1836.11)}{6102.65} - \underset{(0.183)}{0.016} \times \text{fert}$$

6) Explain the meaning of the slope

Increasing fertilizer by one unit decreases gross revenue by 0.016 units. But this reduction is due to chance as there is no systematic relationship between fertilizer and gross revenue. Is this true in the real world?

Section 3 (Stata Outputs, 5 pts)
Note: This one is an open-ended question, as long as your answer was inducted from two regression models and without conceptual mistakes, you will get the full credits.

Compare regression 1 and 2 and explain what you observed

Regression 1: we reject H_0, slope significantly different from zero
Regression 2: we accepted H_0, slope not significantly different from zero