Simple Gravity - Armington Model

ECON 6280 - Introduction of International Economics

Week 1 / Lecture 1 / Note 3

Instructor: Jeffrey Kuo

E-mail: jeffkuo@gwu.edu

Website: http://jeffjkuo.github.io

Lecture Date: June 29, 2020

Modern Trade Model (Armington)

The General Form of Gravity Equation: a statistical model (i.e. atheoretic) which provides a useful mean of organizing the facts of international trade across the countries.

$$X_{ijk} = a_k \frac{Y_i \times Y_j}{D_{ij}}$$

where

- X_{ijk} is the bilateral trade volumes of goods k exported from country i to country j, or the volume of imported goods to country j from country i.
- Be cautious of notation, order of ij matters. Usually $X_{ijk} \neq X_{jik}$
- Y_i is the origin country's GDP, Y_j is the destination country's GDP
- D_{ij} is a "general" term represents the "distance"
- a_k represents the demand shifter of the particular goods, industry k.

Modern Trade Model (Armington) - 2

The Reduced Form of Gravity Equation: a statistical model (i.e. atheoretic) which provides a useful mean of organizing the facts of international trade across the countries.

$$X_{ij} = a\frac{Y_i \times Y_j}{D_{ij}}$$

where

- X_{ij} is the bilateral trade volumes of goods exported from country i to country j, or the volume of imported goods to country j from country i.
- Be cautious of notation, order of ij matters. Usually $X_{ij} \neq X_{ji}$
- Y_i is the origin country's GDP, Y_j is the destination country's GDP
- D_{ij} is a "general" term represents the "distance"
- a represents the demand shifter between trade pair ij. If we aggregate the data into country level, the equation reduce to this.

Import and Market Size

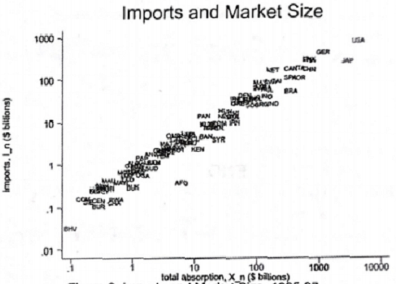

Figure 2: Imports and Market Size, 1995-97

NOTE:
Total absorption: country n's total purchases of manufactures ($X_n = Y_n - E_n + I_n$), an alternative measure of country or market size

Source: EK Ch2

Y- axis: Country n's Imports ($ billions)

X- axis: Total Absorption: Country n's total purchases of manufactures

Bilateral Exports and Production

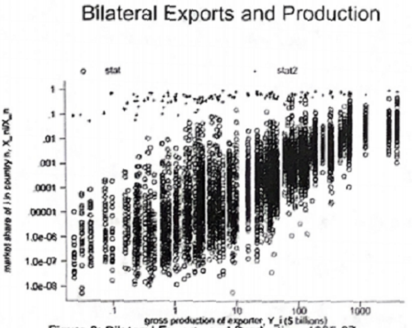

Figure 3: Bilateral Exports and Production, 1995-97

NOTE:
On the y – axes: the share of expenditure on goods from country i in country n (X_{ni}/X_n). There are many export destinations n for each source country i.

ce: EK Ch2

Y- axis: the share of expenditure on goods from country i in country n (X_{ni}/X_n)
X- axis: the gross production of exporter Y_i

Bilateral Trade and Distance II

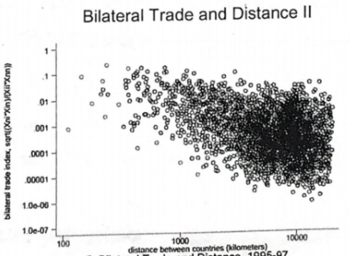

Figure 5: Bilateral Trade and Distance, 1995-97

NOTE:

On the y – axes: Bilateral trade index $B_{ni} = \sqrt{\frac{X_{ni}X_{in}}{X_{ii}X_{nn}}} = \frac{1}{\sqrt{\tau_{ni}\tau_{in}}}$

The slope (the elasticity of trade with respect to trade costs) is -1.17.

Source: EK Ch2

- On the y-axes: Bilateral Trade Index

$$B_{ni} = \sqrt{\frac{X_{ni}X_{in}}{X_{ii}X_{nn}}}$$

- Bilateral Trade Index is negative correlating with the distance

The International Trade of Countries

- The Gravity Equation is successful at explaining a large fraction of the variation om observed bilateral trade flows.
- Geography plays a crucial role in trade pattern.
- This statistical model is not sufficient to answer counterfactual questions:
    - What is the effect of reducing a border tax? A tariff level?
    - Why the relationship is the way it is?
- The Armington model provides the first theoretical foundation for the gravity relationship.

General Equilibrium Model - Demand Side

u_j follows a Constant Elasticity of Substitution (CES) function form

$$u_j = [\sum_{j \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

Representative consumer maximize the demand

$$\max_{q_{ij}} u_j$$

subject to the resource constraint, income of the country Y: Y_j

$$\sum_{j \in S} q_{ij} P_{ij} \leq Y_j$$

set up the lagrangian multiplier euqation

$$L : u_j = [\sum_{j \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} - \lambda[\sum_{j \in S} q_{ij} p_{ij} - Y_j]$$

First Order Condition (F.O.C.)

$$\frac{\partial L}{\partial q_{ij}} = 0$$

General Equilibrium Model - Demand Side

F.O.C with respect to q_{ij}

$$\frac{\sigma}{\sigma-1}[\sum_{j\in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma}(a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{1}{\sigma}}) = \lambda p_{ij}$$

F.O.C with respect to $q_{i'j}$ (different country i')

$$\frac{\sigma}{\sigma-1}[\sum_{j\in S} a_{i'j}^{\frac{1}{\sigma}} q_{i'j}^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma}(a_{i'j}^{\frac{1}{\sigma}} q_{i'j}^{\frac{1}{\sigma}}) = \lambda p_{i'j}$$

Take the ratio of two

$$\frac{a_{ij}}{a_{i'j}} = (\frac{p_{ij}}{p_{i'j}})^{\sigma} \frac{q_{ij}}{q_{i'j}}$$

Algebra, times $p_{i'j}$ at the both hand side,

$$p_{i'j} q_{i'j} = \frac{1}{a_{i'j}} q_{ij} p_{ij}^{\sigma} a_{i'j} p_{i'j}^{1-\sigma}$$

General Equilibrium Model - Demand Side

Algebra, times $p_{i'j}$ at the both hand side,

$$p_{i'j}q_{i'j} = \frac{1}{a_{i'j}} q_{ij} p_{ij}^{\sigma} a_{i'j} p_{i'j}^{1-\sigma}$$

Aggregate up by using $\sum_{i' \in S}$

$$\sum_{i' \in S} p_{i'j}q_{i'j} = \sum_{i' \in S} \frac{1}{a_{i'j}} q_{ij} p_{ij}^{\sigma} a_{i'j} p_{i'j}^{1-\sigma}$$

Identical to, $\sum_{i' \in S}$

$$\sum_{i' \in S} p_{i'j}q_{i'j} = \frac{1}{a_{i'j}} q_{ij} p_{ij}^{\sigma} \sum_{i' \in S} a_{i'j} p_{i'j}^{1-\sigma} \quad (1)$$

Since the preference follow CES form, the Price Index in country j could be shown as,

$$\sum_{i' \in S} a_{i'j} p_{i'j}^{1-\sigma} = P_j^{1-\sigma}$$

General Equilibrium Model - Demand Side

On the other hand, on the left hand side of equation (1),

$$\sum_{i' \in S} p_{i'j} q_{i'j} = Y_j$$

So then the equation (1) reduces to the better form under CES assumptions.

$$Y_j = \frac{1}{a_{ij}} q_{ij} p_{ij}^{\sigma} P_j^{1-\sigma} \tag{2}$$

Algebra,

$$q_{ij} = a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1}$$

By timing p_{ij} on the both hand side, we could have

$$p_{ij} q_{ij} = a_{ij} p_{ij}^{1-\sigma} Y_j P_j^{\sigma-1}$$

Where $p_i j q_{ij} = X_{ij}$ So we got the trade volume with micro foundation

$$X_{ij} = a_{ij} p_{ij}^{1-\sigma} Y_j P_j^{\sigma-1} \tag{3}$$

Equation (3) is the foundation of New-new Trade Theory, which is so-called Armington (Trade) Demand or Gravity Demand.

General Equilibrium (G.E.) Model - Supply Side

Then, we need to figure out the supply side to complete the G.E. Model. Note the supply side could be solved under following three assumptions.

1. Under Perfect competition (Ricardian Spirits)
2. Under the endowment constriants (H-O theorem spirits)
3. Under Monopolistic Competition (New Trade Theory, Krugman Preference)

Here we starts with the simplest one, under the perfect competition (PC) assumptions. Due to PC assumption, $p = mc$

$$mc_{ij} = \frac{w_i \tau_{ij}}{A_i} = p_{ij} \tag{4}$$

where A_{ij} is the linear form of technology τ_{ij} is the trade resistance between ij, could be tariff, distance, or shipping cost.

$\star$ Note τ_{ij} in trade literature are very often to denote as the iceberg shipping cost, and usually $\tau_{ij} > 1$. It means, to make one unit of goods to the destination, you need to ship τ unit of goods. Goods "melt" or perish on the way during shipping.

General Equilibrium (G.E.) Model - Solution

Combine Demand from (3) and Supply from (4), we can solve out the closed form solution of Armington Model under Perfect Competition assumptions.

$$X_{ij} = a_{ij} p_{ij}^{1-\sigma} Y_j P_j^{\sigma-1} \tag{3}$$

$$mc_{ij} = \frac{w_i \tau_{ij}}{A_i} = p_{ij} \tag{4}$$

Plug (4) into (3),

$$X_{ij} = a_{ij} \frac{w_i \tau_{ij}}{A_i}^{1-\sigma} Y_j P_j^{\sigma-1}$$

Does it show the spirits of Gravity Model? What do you see?